Spatially Varying Autoregressive Models for Prediction of New HIV Diagnoses-Supplement

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1 Exploratory Analysis

1.1 Normality Assumptions

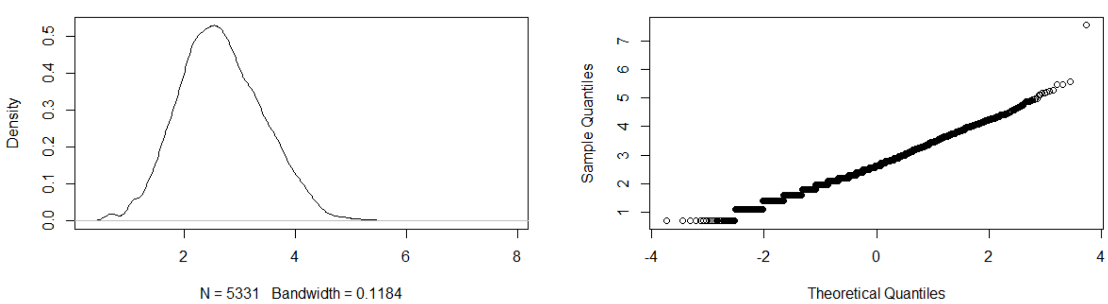

Figure 1: Density plot (left) and QQ normality plot (right) of $\log(Y_{i,t})$ of all available county data in the US from 2008 to 2014.

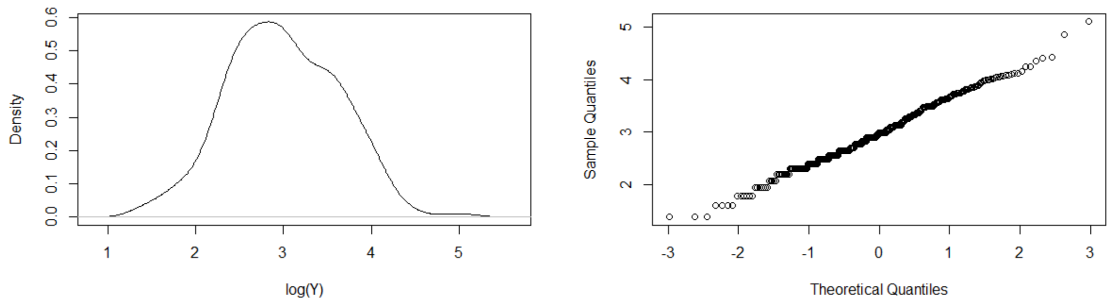

Figure 2: Density plot (left) and QQ normality plot (right) of $\log(Y_{i,t})$ of all available county data in Florida from 2008 to 2014.

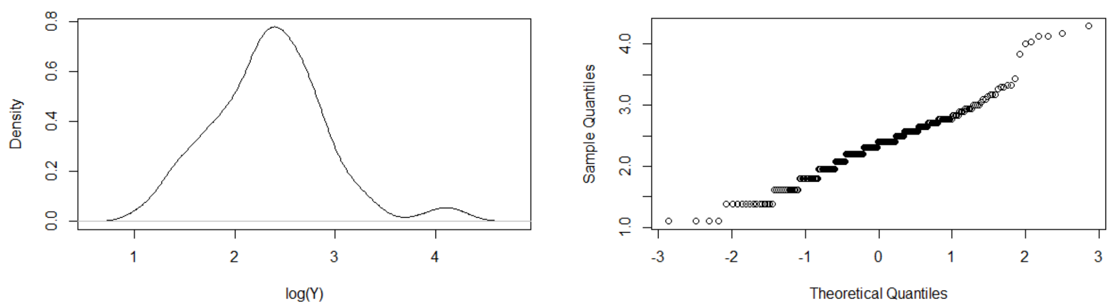

Figure 3: Density plot (left) and QQ normality plot (right) of $\log(Y_{i,t})$ of all available county data in California from 2008 to 2014.

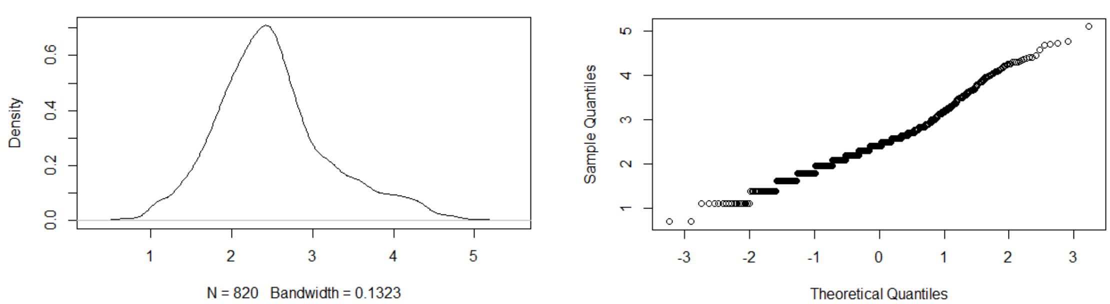

Figure 4: Density plot (left) and QQ normality plot (right) of $\log(Y_{i,t})$ of all available county data in New England from 2008 to 2014.

1.2 Population Size

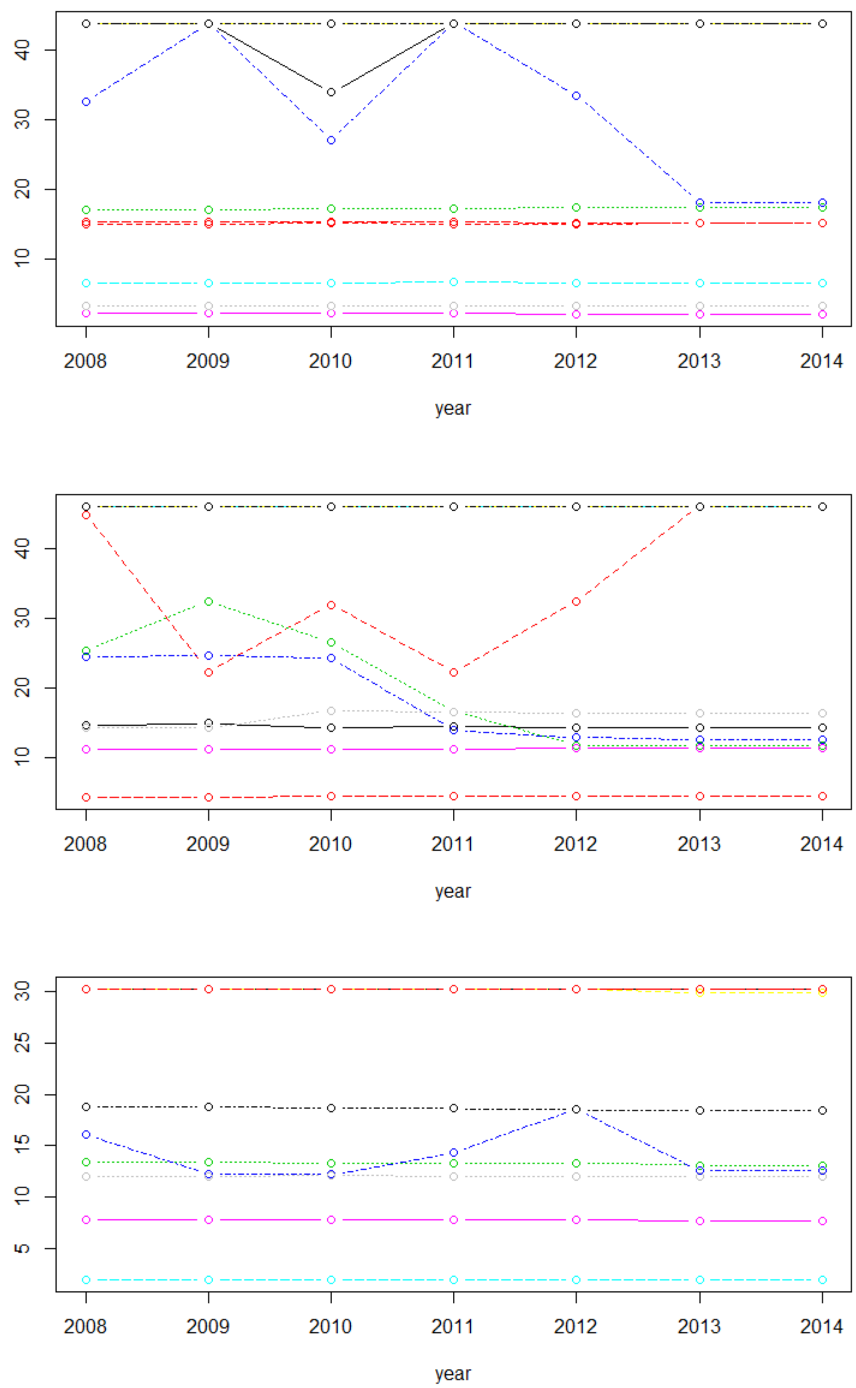

Figure 5: $q_{i,t} = Y_{i,t} n_{i,t}/c$ of a random selection of 10 counties from Florida (top), California(middle) and North England(bottom).

2 Derivation of Q

Define $Y'_{i,t}$ to be the number of HIV new diagnosis cases and $Y_{i,t}$ the number of cases per 100,000 people in county i at year t. Consider the traditional disease modeling framework with first level of the BHM specified as

$$Y'_{i,t} \sim \text{Poisson}(n_{i,t} r_{i,t})$$

where $n_{i,t}$ and $r_{i,t}$ denote the population size and rate of cases per person respectively. Then, as $n_{i,t} \to \infty$,

$$Y'_{i,t} \xrightarrow{D} \text{N}(n_{i,t} r_{i,t}, n_{i,t} r_{i,t}).$$

and since $Y_{i,t} = cY'_{i,t}/n_{i,t},\ c = 100,000,$

$$Y_{i,t} \xrightarrow{D} \text{N}\big(cr_{i,t}, \frac{c^2 r_{i,t}}{n_{i,t}}\big).$$

Transforming $Y_{i,t}$ by taking the natural log, we have

$$\log(Y_{i,t}) - \log(cr_{i,t}) \xrightarrow{D} \text{N}\big(0, \frac{1}{n_{i,t} r_{i,t}}\big)$$

by the delta method. Lastly, we can estimate $n_{it} r_{i,t}$ with $Y'_{i,t} = Y_{i,t} n_{i,t}/c$ to obtain,

$$\log(Y_{i,t}) - \log(cr_{i,t}) \xrightarrow{D} \text{N}\big(0, \frac{c}{n_{i,t} Y_{i,t}}\big).$$

So the variance of $\log Y_{i,t}$ is inversely proportional to $n_{i,t} Y_{i,t}$.

3 Full conditional distributions and MH algorithm details.

Define $Z_{i,t} = log(Y_{i,t})$ to be the detrended log transformed new HIV diagnoses rates for $i = 1, \ldots, n,\ t = 1, \ldots, T$ where n and T are the number of available counties and years respectively.

Normal and Inverse Gamma Distributions

Normal Distribution: $f(x; \mu, \Sigma) = \frac{1}{2\pi|\Sigma|^{1/2}} exp(-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu))$

Gamma Distribution: $f(x; a, b) = \frac{b^a}{\Gamma(a)} x^{-a-1} exp(-\frac{b}{x})$

3.1 Model 6

Here we derive all full conditional distributions and Metropolis-Hastings (MH) algorithm details for model 6. Full conditionals for all submodels(1-5) follow a similar form.

Level I.

$$Z_{i,t} = X_{i,t}\boldsymbol{\beta} + \alpha_t + \phi_i + \delta_{i,t} + \epsilon_{i,t},\ \epsilon \sim N(0, \sigma^2 Q),\quad Q = \text{diag}\big(\frac{c}{Y_{i,t} n_{i,t}}\big)$$

Level II (Priors).

$$\boldsymbol{\beta}|\tau_\beta^2 \overset{iid}{\sim} N(0,\tau_\beta^2)$$

$$\boldsymbol{\alpha} \sim N(0,\Sigma_\alpha),\ \boldsymbol{\phi} \sim N(0,\Sigma_\phi),\ \boldsymbol{\delta} \sim N(0,\Sigma_\delta)$$
$$\Sigma_\alpha = \tau_\alpha^2 \rho_\alpha^{|u|},\ \Sigma_\phi = \tau_\phi^2[(1-\lambda_\phi)I + \lambda_\phi R]^{-1},\ \Sigma_\delta = \tau_\delta^2 \rho_\delta^{|u|} \otimes [(1-\lambda_\delta)I + \lambda_\delta R]^{-1}$$

$$\mu_Z = E[\mathbf{Z}|\mu,\boldsymbol{\beta},\boldsymbol{\alpha},\boldsymbol{\phi},\boldsymbol{\delta}] = X\boldsymbol{\beta} + \boldsymbol{\alpha} + \boldsymbol{\phi} + \boldsymbol{\delta},\ \Sigma_Z = \sigma^2 Q$$

Hyperpriors.

$$\tau_\alpha^2,\ \tau_\phi^2,\ \tau_\delta^2 \sim IG(a_1,b_1),\ \rho_\alpha,\ \rho_\delta \sim Uniform(-1,1),\ \lambda_\phi,\ \lambda_\delta \sim Uniform[0,1],\ \sigma^2 \sim IG(a_2,b_2)$$

Model 6 Full conditional distributions

Full conditional distribution of $\boldsymbol{\beta}$:

Given prior $\boldsymbol{\beta}|\tau_\beta^2 \overset{iid}{\sim} N(0,\tau_\beta^2)$, $[\boldsymbol{\beta}|\mathbf{Z},\cdot] \propto [\mathbf{Z}|\boldsymbol{\beta},\cdot][\boldsymbol{\beta}|\tau_\beta^2]$

$\propto exp\big\{-\frac{1}{2\sigma^2}(\mathbf{Z} - X\boldsymbol{\beta} - \boldsymbol{\alpha} - \boldsymbol{\phi} - \boldsymbol{\delta})'Q^{-1}(\mathbf{Z} - X\boldsymbol{\beta} - \boldsymbol{\alpha} - \boldsymbol{\phi} - \boldsymbol{\delta})\big\} \times exp\big\{-\frac{1}{2\tau_\beta^2}\boldsymbol{\beta}'\boldsymbol{\beta}\big\}$

$\propto exp\big\{-\frac{1}{2}[\boldsymbol{\beta}'[-\frac{1}{2\sigma^2}X'Q^{-1}X + \frac{1}{\tau_\beta^2}]\beta - \beta'[\frac{1}{\sigma^2}X'Q^{-1}(\mathbf{Z} - \boldsymbol{\alpha} - \boldsymbol{\phi} - \boldsymbol{\delta})]$
$-[\frac{1}{\sigma^2}(\mathbf{Z} - \boldsymbol{\alpha} - \boldsymbol{\phi} - \boldsymbol{\delta})Q^{-1}X]\boldsymbol{\beta}\big\}$

by completing the square, we have the full conditional

$$\boldsymbol{\beta}|\mathbf{Z},\cdot \sim MVN(\mu_\beta,\Sigma_\beta)$$

$\Sigma_\beta = [\frac{1}{\sigma^2}X'Q^{-1}X + \frac{1}{\tau_\beta^2}]^{-1}$
$\mu_\beta = \Sigma_\beta \frac{1}{\sigma^2}X'Q^{-1}(\mathbf{Z} - \boldsymbol{\alpha} - \boldsymbol{\phi} - \boldsymbol{\delta})$

Full conditional distribution of $\boldsymbol{\alpha}$:

Given prior $\boldsymbol{\alpha}|\rho_\alpha,\tau_\alpha^2 \sim N(0,\Sigma_\alpha)$, $[\boldsymbol{\alpha}|\mathbf{Z},\cdot] \propto [\mathbf{Z}|\boldsymbol{\alpha},\cdot][\boldsymbol{\alpha}|\tau_\alpha^2,\rho]$

$\propto exp\big\{-\frac{1}{2\sigma^2}(\mathbf{Z} - X\boldsymbol{\beta} - \boldsymbol{\alpha}\otimes 1_n - 1_T\otimes\boldsymbol{\phi} - \boldsymbol{\delta})'Q^{-1}(\mathbf{Z} - X\boldsymbol{\beta} - \boldsymbol{\alpha}\otimes 1_n - 1_T\otimes\boldsymbol{\phi} - \boldsymbol{\delta})\big\}$
$\times exp\big\{-\frac{1}{2}\boldsymbol{\alpha}'\Sigma_\alpha^{-1}\boldsymbol{\alpha}\big\}$

$\propto exp\big\{-\frac{1}{2}[\boldsymbol{\alpha}'[\frac{Q_1^{-1}}{\sigma^2}+\Sigma_\alpha^{-1}]\boldsymbol{\alpha}-\boldsymbol{\alpha}'\frac{1}{2\sigma^2}Q_2^{-1}(\mathbf{Z}-X\boldsymbol{\beta}-1_T\otimes\boldsymbol{\phi}-\boldsymbol{\delta})-\frac{1}{2\sigma^2}(\mathbf{Z}-X\boldsymbol{\beta}-1_T\otimes\boldsymbol{\phi}-\boldsymbol{\delta})^TQ_2^{-1}\boldsymbol{\alpha}]\big\}$

Q_1 is a $T\times T$ diagonal matrix with t^{th} diagonal entry as $\Sigma_{i=1}^n\frac{Y_{i,t}n_{i,t}}{c}$ and

Q_2 is a concatenation of n $T\times T$ diagonal matrices where the t^{th} diagonal entry of the i^{th} matrix is $\frac{Y_{i,t}n_{i,t}}{c}$.

The full conditional distribution becomes:

$$\boldsymbol{\alpha}|\mathbf{Z},\cdot\sim MVN(\mu_\alpha,\Sigma_\alpha^{**})$$

$\Sigma_\alpha^{**}=[\frac{Q_1^{-1}}{\sigma^2}+\Sigma_\alpha^{-1}]^{-1},\ \mu_\alpha=\Sigma_\alpha^{**}Q_2^{-1}\frac{1}{\sigma^2}(\mathbf{Z}-X\boldsymbol{\beta}-1_T\otimes\boldsymbol{\phi}-\boldsymbol{\delta})$

Full conditional distribution of $\boldsymbol{\phi}$:

Given prior $\boldsymbol{\phi}|\lambda_\phi,\tau_\phi^2\sim N(0,\Sigma_\phi)$, $[\boldsymbol{\phi}|\mathbf{Z},\cdot]\propto[\mathbf{Z}|\boldsymbol{\phi},\cdot][\boldsymbol{\phi}|\tau_\phi^2,\lambda_\phi]$

$\propto exp\big\{-\frac{1}{2\sigma^2}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-1_T\otimes\boldsymbol{\phi}-\boldsymbol{\delta})'Q^{-1}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-1_T\otimes\boldsymbol{\phi}-\boldsymbol{\delta})\big\}$
$\times exp\big\{-\frac{1}{2}\boldsymbol{\phi}'\Sigma_\phi^{-1}\boldsymbol{\phi}\big\}$

$\propto exp\big\{-\frac{1}{2}[\boldsymbol{\phi}'[\frac{Q_3^{-1}}{\sigma^2}+\Sigma_\phi^{-1}]\boldsymbol{\phi}-\boldsymbol{\phi}'Q_4^{-1}\frac{1}{\sigma^2}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-\boldsymbol{\delta})$
$-\frac{1}{\sigma^2}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-\boldsymbol{\delta})Q_4^{-1}\boldsymbol{\phi}]\big\}$

Q_3 is a $n\times n$ diagonal matrix with i^{th} diagonal entry as $\Sigma_{t=1}^T\frac{Y_{i,t}n_{i,t}}{c}$ and

Q_4 is a concatenation of T $n\times n$ diagonal matrices where the i^{th} diagonal entry of the t^{th} matrix is $\frac{Y_{i,t}n_{i,t}}{c}$.

The full conditional distribution becomes:

$$\boldsymbol{\phi}|\mathbf{Z},\cdot\sim MVN(\mu_\phi,\Sigma_\phi^{**})$$

$\Sigma_\phi^{**}=[\frac{Q_3^{-1}}{\sigma^2}+\Sigma_\phi^{-1}]^{-1},\ \mu_\phi=\Sigma_\phi^{**}\frac{1}{\sigma^2}Q_4^{-1}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-\boldsymbol{\delta})$

Full conditional distribution of $\boldsymbol{\delta}$:

Given prior $\boldsymbol{\delta}|\rho_\delta,\lambda_\delta,\tau_\delta^2\sim N(0,\Sigma_\delta)$, $[\boldsymbol{\delta}|\mathbf{Z},\cdot]\propto[\mathbf{Z}|\boldsymbol{\delta},\cdot][\boldsymbol{\delta}|\tau_\delta^2,\lambda_\delta,\rho_\delta]$

$\propto exp\big\{-\frac{1}{2\sigma^2}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-1_T\otimes\boldsymbol{\phi}-\boldsymbol{\delta})'Q^{-1}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-1_T\otimes\boldsymbol{\phi}-\boldsymbol{\delta})\big\}$
$\times exp\big\{-\frac{1}{2}\boldsymbol{\delta}'\Sigma_\delta^{-1}\boldsymbol{\delta}\big\}$

$\propto exp\big\{-\frac{1}{2}[\boldsymbol{\delta}'[\frac{1}{\sigma^2}+\Sigma_\delta^{-1}]\boldsymbol{\delta}-\boldsymbol{\delta}'[\frac{1}{\sigma^2}Q^{-1}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-1_T\otimes\boldsymbol{\phi})]$
$-\frac{1}{\sigma^2}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-1_T\otimes\boldsymbol{\phi})'Q^{-1}\boldsymbol{\delta}\big\}$

The full conditional distribution becomes:

$$\boldsymbol{\delta}|\mathbf{Z},\cdot\sim MVN(\mu_\delta,\Sigma_\delta^{**})$$

where $\Sigma_\delta^{**}=[\frac{Q^{-1}}{\sigma^2}+\Sigma_\delta^{-1}]^{-1},\ \mu_\delta=\Sigma_\delta^{**}\frac{1}{\sigma^2}Q^{-1}(\mathbf{Z}-X\boldsymbol{\beta}-\boldsymbol{\alpha}\otimes 1_n-1_T\otimes\boldsymbol{\phi})$

Full conditional distribution of σ^2 :

Given prior $\sigma^2|a_2,b_2\sim IG(a_2,b_2)$, $[\sigma^2|\mathbf{Z},\cdot]\propto[\mathbf{Z}|\sigma^2,\cdot][\sigma^2|a_2,b_2]$

$\propto(\sigma^2)^{-nT/2}exp\big\{-\frac{1}{2\sigma^2}(\mathbf{Z}-\mu_Z)'Q^{-1}(\mathbf{Z}-\mu_Z)\big\}\times(\sigma^2)^{-a_2-1}exp\big\{-\frac{b_2}{\sigma^2}\big\}$

$\propto(\sigma^2)^{-(nT/2+a_2)-1}exp\big\{-\frac{1}{\sigma^2}\big[\frac{1}{2}(\mathbf{Z}-\mu_Z)'Q^{-1}(\mathbf{Z}-\mu_Z)+b_2\big]\big\}$

The full conditional distribution becomes:

$$\sigma^2|\mathbf{Z},\cdot\sim IG\big(\frac{nT}{2}+a_2,\frac{1}{2}(\mathbf{Z}-\mu_Z)'Q^{-1}(\mathbf{Z}-\mu_Z)+b_2\big)$$

Full conditional distribution of τ_α^2 :
Let $\Sigma_\alpha=\tau_\alpha^2\Sigma_\alpha^*$

Given prior $\tau_\alpha^2|a_1,b_1\sim IG(a_1,b_1)$, $[\tau_\alpha^2|\mathbf{Z},\boldsymbol{\alpha},\cdot]\propto[\boldsymbol{\alpha}|\tau_\alpha^2,\rho_\alpha][\tau_\alpha^2|a_1,b_1]$

$\propto(\tau_\alpha^2)^{-T/2}exp\{-\frac{1}{2\tau_\alpha^2}\boldsymbol{\alpha}'\Sigma_\alpha^*\boldsymbol{\alpha}\}\times(\tau_\alpha^2)^{-a_1-1}exp\big\{-\frac{b_1}{\tau_\alpha^2}\big\}$

$\propto(\tau_\alpha^2)^{-(T/2+a_1)-1}exp\big\{-\frac{1}{\tau_\alpha^2}\big[\frac{1}{2}\boldsymbol{\alpha}'\Sigma_\alpha^*\boldsymbol{\alpha}+b_1\big]\big\}$

The full conditional distribution becomes:

$$\tau_\alpha^2|\mathbf{Z},\boldsymbol{\alpha},\cdot\sim \mathrm{IG}\big(\frac{T}{2}+a_1,\frac{1}{2}\boldsymbol{\alpha}'\Sigma_\alpha^*\boldsymbol{\alpha}+b_1\big)$$

Full conditional distribution of τ_ϕ^2 :

Let $\Sigma_\phi=\tau_\phi^2\Sigma_\phi^*$

Given prior $\tau_\phi^2|a_1,b_1\sim IG(a_1,b_1)$, $[\tau_\phi^2|\mathbf{Z},\boldsymbol{\phi},\cdot]\propto[\boldsymbol{\phi}|\tau_\phi^2,\lambda_\phi][\tau_\phi^2|a_1,b_1]$

$\propto (\tau_\phi^2)^{-n/2} exp\{-\frac{1}{2\tau_\phi^2}\boldsymbol{\phi}'\Sigma_\phi^*\boldsymbol{\phi}\} \times (\tau_{\boldsymbol{\phi}}^2)^{-a_1-1} exp\{-\frac{b_1}{\tau_\phi^2}\}$

$\propto (\tau_\phi^2)^{-(n/2+a_1)-1} exp\{-\frac{1}{\tau_\phi^2}\big[\frac{1}{2}\boldsymbol{\phi}'\Sigma_\phi^*\boldsymbol{\phi} + b_1\big]\}$

The full conditional distribution becomes:

$$\tau_\phi^2|\mathbf{Z}, \boldsymbol{\phi}, \cdot \sim \text{IG}\big(\frac{n}{2} + a_1, \frac{1}{2}\boldsymbol{\phi}'\Sigma_\phi^*\boldsymbol{\phi} + b_1\big)$$

Full conditional distribution of τ_δ^2 :
Let $\Sigma_\delta = \tau_\delta^2\Sigma_\delta^*$

Given prior $\tau_\delta^2|a_1, b_1 \sim IG(a_1, b_1)$, $[\tau_\delta^2|\mathbf{Z}, \boldsymbol{\delta}, \cdot] \propto [\boldsymbol{\delta}|\tau_\delta^2, \lambda_\delta, \rho_\delta][\tau_\delta^2|a_1, b_1]$

$\propto (\tau_\delta^2)^{-nT/2} exp\{-\frac{1}{2\tau_\delta^2}\boldsymbol{\delta}'\Sigma_\delta^*\boldsymbol{\delta}\} \times (\tau_\delta^2)^{-a_1-1} exp\{-\frac{b_1}{\tau_\delta^2}\}$

$\propto (\tau_\delta^2)^{-(nT/2+a_1)-1} exp\{-\frac{1}{\tau_\delta^2}\big[\frac{1}{2}\boldsymbol{\delta}'\Sigma_\delta^*\boldsymbol{\delta} + b_1\big]\}$

The full conditional distribution becomes:

$$\tau_\delta^2|\mathbf{Z}, \boldsymbol{\delta}, \cdot \sim \text{IG}\big(\frac{nT}{2} + a_1, \frac{1}{2}\boldsymbol{\delta}'\Sigma_\delta^*\boldsymbol{\delta} + b_1\big)$$

Model 6 MH sampling algorithms

Sampling ρ_α at iteration k :

1. Draw ρ_α^* from proposal $q(\rho_\alpha^*|\rho_\alpha^{(k-1)}) \sim N(\rho_\alpha^{(k-1)}, s_\alpha^2)\mathbb{1}\{0 \le \rho_\alpha^* \le 1\}$.

   where $q(\rho_\alpha^*|\rho_\alpha^{(k-1)}) = \frac{\phi\left(\frac{\rho_\alpha^*-\rho_\alpha^{(k-1)}}{s_\alpha}\right)}{\Phi\left(\frac{1-\rho_\alpha^{(k-1)}}{s_\alpha}\right)-\Phi\left(\frac{-\rho_\alpha^{(k-1)}}{s_\alpha}\right)}$.

2. Calculate acceptance ratio $r_\alpha = \frac{p(\rho_\alpha^*|\cdot)q(\rho_\alpha^{(k-1)}|\rho_\alpha^*)}{p(\rho_\alpha^{(k-1)}|\cdot)q(\rho_\alpha^*|\rho_\alpha^{(k-1)})}$

$$\propto \frac{|\Sigma_\alpha(\rho_\alpha^*)|^{-1/2} exp\{-\frac{1}{2}\boldsymbol{\alpha}'\Sigma_\alpha^{-1}(\rho_\alpha^*)\boldsymbol{\alpha}\}\left[\Phi\left(\frac{1-\rho_\alpha^{(k-1)}}{s_\alpha}\right)-\Phi\left(\frac{-1-\rho_\alpha^{(k-1)}}{s_\alpha}\right)\right]}{|\Sigma_\alpha(\rho_\alpha^{(k-1)})|^{-1/2} exp\{-\frac{1}{2}\boldsymbol{\alpha}'\Sigma_\alpha^{-1}(\rho_\alpha^{(k-1)})\boldsymbol{\alpha}\}\left[\Phi\left(\frac{1-\rho_\alpha^*}{s_\alpha}\right)-\Phi\left(\frac{-1-\rho_\alpha^*}{s_\alpha}\right)\right]}$$

$$= \left[\frac{|\Sigma_\alpha(\rho_\alpha^*)|}{|\Sigma_\alpha(\rho_\alpha^{(k-1)})|}\right]^{-1/2} exp\Big\{-\frac{1}{2}\boldsymbol{\alpha}'[\Sigma_\alpha^{-1}(\rho_\alpha^*) - \Sigma_\alpha^{-1}(\rho_\alpha^{(k-1)})]\boldsymbol{\alpha}\Big\}\left[\frac{\Phi\left(\frac{1-\rho_\alpha^{(k-1)}}{s_\alpha}\right)-\Phi\left(\frac{-1-\rho_\alpha^{(k-1)}}{s_\alpha}\right)}{\Phi\left(\frac{1-\rho_\alpha^*}{s_\alpha}\right)-\Phi\left(\frac{-1-\rho_\alpha^*}{s_\alpha}\right)}\right]$$

3. Generate $\xi \sim Uniform(0,1)$.
$\rho_\alpha^{(k)} = \begin{cases} \rho_\alpha^* & if\ r_\alpha > \xi \text{ (with acceptance rate} = min\{1, r_\alpha\}), \\ \rho_\alpha^{(k-1)} & o.w. \end{cases}$

Sampling λ_ϕ at iteration k :

1. Draw λ_ϕ^* from proposal $q(\lambda_\phi^*|\lambda_\phi^{(k-1)}) \sim N(\lambda_\phi^{(k-1)}, s_\phi^2)\mathbb{1}\{0 \le \lambda_\phi^* \le 1\}$.

   where $q(\lambda_\phi^*|\lambda_\phi^{(k-1)}) = \dfrac{\phi\Big(\frac{\lambda_\phi^* - \lambda_\phi^{(k-1)}}{s_\phi}\Big)}{\Phi\Big(\frac{1-\lambda_\phi^{(k-1)}}{s_\phi}\Big) - \Phi\Big(\frac{-\lambda_\phi^{(k-1)}}{s_\phi}\Big)}$.

2. Calculate acceptance ratio $r_\phi = \frac{p(\lambda_\phi^*|\cdot)q(\lambda_\phi^{(k-1)}|\lambda_\phi^*)}{p(\lambda_\phi^{(k-1)}|\cdot)q(\lambda_\phi^*|\lambda_\phi^{(k-1)})}$

$$\propto \frac{|\Sigma_\phi(\lambda_\phi^*)|^{-1/2} exp\{-\frac{1}{2}\boldsymbol{\phi}'\Sigma_\phi^{-1}(\lambda_\phi^*)\boldsymbol{\phi}\}\Big[\Phi\Big(\frac{1-\lambda_\phi^{(k-1)}}{s_\phi}\Big) - \Phi\Big(\frac{-\lambda_\phi^{(k-1)}}{s_\phi}\Big)\Big]}{|\Sigma_\phi(\lambda_\phi^{(k-1)})|^{-1/2} exp\{-\frac{1}{2}\boldsymbol{\phi}'\Sigma_\phi^{-1}(\lambda_\phi^{(k-1)})\boldsymbol{\phi}\}\Big[\Phi\Big(\frac{1-\lambda_\phi^*}{s_\phi}\Big) - \Phi\Big(\frac{-\lambda_\phi^*}{s_\phi}\Big)\Big]}$$

$$= \Bigg[\frac{|\Sigma_\phi(\lambda_\phi^*)|}{|\Sigma_\phi(\lambda_\phi^{(k-1)})|}\Bigg]^{-1/2} exp\Big\{-\frac{1}{2}\boldsymbol{\phi}'[\Sigma_\phi^{-1}(\lambda_\phi^*) - \Sigma_\phi^{-1}(\lambda_\phi^{(k-1)})]\boldsymbol{\phi}\Big\}\Bigg[\frac{\Phi\Big(\frac{1-\lambda_\phi^{(k-1)}}{s_\phi}\Big) - \Phi\Big(\frac{-\lambda_\phi^{(k-1)}}{s_\phi}\Big)}{\Phi\Big(\frac{1-\lambda_\phi^*}{s_\phi}\Big) - \Phi\Big(\frac{-\lambda_\phi^*}{s_\phi}\Big)}\Bigg]$$

3. Generate $\xi \sim Uniform(0,1)$.
$\lambda_\phi^{(k)} = \begin{cases} \lambda_\phi^* & if\ r_\phi > \xi \text{ (with acceptance rate} = min\{1, r_\phi\}), \\ \lambda_\phi^{(k-1)} & o.w. \end{cases}$

Sampling ρ_δ at iteration k :

1. Draw ρ_δ^* from proposal $q(\rho_\delta^*|\rho_\delta^{(k-1)}) \sim N(\rho_\delta^{(k-1)}, s_\delta^2)\mathbb{1}\{0 \le \rho_\delta^* \le 1\}$.

   where $q(\rho_\delta^*|\rho_\delta^{(k-1)}) = \dfrac{\phi\Big(\frac{\rho_\delta^* - \rho_\delta^{(k-1)}}{s_\delta}\Big)}{\Phi\Big(\frac{1-\rho_\delta^{(k-1)}}{s_\delta}\Big) - \Phi\Big(\frac{-\rho_\delta^{(k-1)}}{s_\delta}\Big)}$.

2. Calculate acceptance ratio $r_\delta = \frac{p(\rho_\delta^*|\cdot)q(\rho_\delta^{(k-1)}|\rho_\delta^*)}{p(\rho_\delta^{(k-1)}|\cdot)q(\rho_\delta^*|\rho_\delta^{(k-1)})}$

$$\propto \frac{|\Sigma_\delta(\rho_\delta^*)|^{-1/2} exp\{-\frac{1}{2}\boldsymbol{\delta}'\Sigma_\delta^{-1}(\rho_\delta^*)\boldsymbol{\delta}\}\Big[\Phi\Big(\frac{1-\rho_\delta^{(k-1)}}{s_\delta}\Big) - \Phi\Big(\frac{-1-\rho_\delta^{(k-1)}}{s_\delta}\Big)\Big]}{|\Sigma_\delta(\rho_\delta^{(k-1)})|^{-1/2} exp\{-\frac{1}{2}\boldsymbol{\delta}'\Sigma_\delta^{-1}(\rho_\delta^{(k-1)})\boldsymbol{\delta}\}\Big[\Phi\Big(\frac{1-\rho_\delta^*}{s_\delta}\Big) - \Phi\Big(\frac{-1-\rho_\delta^*}{s_\delta}\Big)\Big]}$$

$$= \Bigg[\frac{|\Sigma_\delta(\rho_\delta^*)|}{|\Sigma_\delta(\rho_\delta^{(k-1)})|}\Bigg]^{-1/2} exp\Big\{-\frac{1}{2}\boldsymbol{\delta}'[\Sigma_\delta^{-1}(\rho_\delta^*) - \Sigma_\delta^{-1}(\rho_\delta^{(k-1)})]\boldsymbol{\delta}\Big\}\Bigg[\frac{\Phi\Big(\frac{1-\rho_\delta^{(k-1)}}{s_\delta}\Big) - \Phi\Big(\frac{-1-\rho_\delta^{(k-1)}}{s_\delta}\Big)}{\Phi\Big(\frac{1-\rho_\delta^*}{s_\delta}\Big) - \Phi\Big(\frac{-1-\rho_\delta^*}{s_\delta}\Big)}\Bigg]$$

3. Generate $\xi \sim Uniform(0,1)$.
$\rho_\delta^{(k)} = \begin{cases} \rho_\delta^* & if\ r_\delta > \xi \text{ (with acceptance rate} = min\{1, r_\delta\}), \\ \rho_\delta^{(k-1)} & o.w. \end{cases}$

Sampling λ_δ at iteration k :

1. Draw λ_δ^* from proposal $q(\lambda_\delta^*|\lambda_\delta^{(k-1)}) \sim N(\lambda_\delta^{(k-1)}, s_{\delta 1}^2)\mathbb{1}\{0 \leq \lambda_\delta^* \leq 1\}$.

   where $q(\lambda_\delta^*|\lambda_\delta^{(k-1)}) = \frac{\phi\left(\frac{\lambda_\delta^* - \lambda_\delta^{(k-1)}}{s_{\delta 1}}\right)}{\Phi\left(\frac{1-\lambda_\delta^{(k-1)}}{s_{\delta 1}}\right) - \Phi\left(\frac{-\lambda_\delta^{(k-1)}}{s_{\delta 1}}\right)}$.

2. Calculate acceptance ratio $r_{\delta 1} = \frac{p(\lambda_\delta^*|\cdot)q(\lambda_\delta^{(k-1)}|\lambda_\delta^*)}{p(\lambda_\delta^{(k-1)}|\cdot)q(\lambda_\delta^*|\lambda_\delta^{(k-1)})}$

$$\propto \frac{|\Sigma_\delta(\lambda_\delta^*)|^{-1/2} exp\{-\frac{1}{2}\boldsymbol{\delta}'\Sigma_\delta^{-1}(\lambda_\delta^*)\boldsymbol{\delta}\}\left[\Phi\left(\frac{1-\lambda_\delta^{(k-1)}}{s_{\delta 1}}\right) - \Phi\left(\frac{-\lambda_\delta^{(k-1)}}{s_{\delta 1}}\right)\right]}{|\Sigma_\delta(\lambda_\delta^{(k-1)})|^{-1/2} exp\{-\frac{1}{2}\boldsymbol{\delta}'\Sigma_\delta^{-1}(\lambda_\delta^{(k-1)})\boldsymbol{\delta}\}\left[\Phi\left(\frac{1-\lambda_\delta^*}{s_{\delta 1}}\right) - \Phi\left(\frac{-\lambda_\delta^*}{s_{\delta 1}}\right)\right]}$$

$$= \left[\frac{|\Sigma_\delta(\lambda_\delta^*)|}{|\Sigma_\delta(\lambda_\delta^{(k-1)})|}\right]^{-1/2} exp\Big\{-\frac{1}{2}\boldsymbol{\delta}'[\Sigma_\delta^{-1}(\lambda_\delta^*) - \Sigma_\phi^{-1}(\lambda_\delta^{(k-1)})]\boldsymbol{\delta}\Big\}\left[\frac{\Phi\left(\frac{1-\lambda_\delta^{(k-1)}}{s_{\delta 1}}\right) - \Phi\left(\frac{-\lambda_\delta^{(k-1)}}{s_{\delta 1}}\right)}{\Phi\left(\frac{1-\lambda_\delta^*}{s_{\delta 1}}\right) - \Phi\left(\frac{-\lambda_\delta^*}{s_{\delta 1}}\right)}\right]$$

3. Generate $\xi \sim Uniform(0,1)$.
$\lambda_\delta^{(k)} = \begin{cases} \lambda_\delta^* & if\ r_{\delta 1} > \xi \text{ (with acceptance rate} = min\{1, r_{\delta 1}\}), \\ \lambda_\delta^{(k-1)} & o.w. \end{cases}$

3.2 SVAR Model

Here we derive all full conditional distributions and MH sampling algorithm details for the SVAR model. Models 7 and 8 are special cases of this model.

Level I.

$$Z_{i,t} = X_{i,t-1}\boldsymbol{\beta} + \psi_{i,t-1}\rho_i(Z_{i,t-1} - X_{i,t-2}\boldsymbol{\beta}) + \epsilon_{i,t}\ \boldsymbol{\epsilon} \overset{iid}{\sim} N(0, \sigma^2 N)$$

$$\mu_Z = E[\mathbf{Z}] = X\boldsymbol{\beta},\ Cov(Z_{\cdot,t}, Z_{\cdot,t-u}) = \begin{cases} \frac{c}{\sqrt{Y_{i,t}n_{i,t}Y_{i,t-u}n_{i,t-u}}}\frac{\sigma^2\boldsymbol{\rho}^{|u|}}{1-\boldsymbol{\rho}^2} & h = 0 \\ 0 & o.w. \end{cases}$$

$$f(\mathbf{Z}|\boldsymbol{\beta}, \boldsymbol{\rho}, \sigma^2) = (2\pi)^{-nT/2}|\Sigma_Z|^{-1/2} exp\Big\{-\frac{1}{2}(Z - \mu_Z)'\Sigma_Z^{-1}(Z - \mu_Z)\Big\}$$

Level II (Priors).

$$\boldsymbol{\beta}|\tau_\beta^2 \overset{iid}{\sim} N(0, \tau_\beta^2)$$

$$\rho_i = \Phi(W_i) \text{ such that } \rho_i \in (-1,1) \text{ where } \mathbf{W} \sim N(0, \tau_\rho^2[(1-\lambda_\rho)I + \lambda_\rho R]^{-1})$$

$\boldsymbol{\rho}$ is defined through the copula density $g(u_1,...,u_n) = \frac{\phi_{\Sigma_W}(F_1^{-1}(u_1),...,F_n^{-1}(u_n))}{\prod_{i=1}^n f_i(F_i^{-1}(u_i))}$

$= \frac{\phi_{\Sigma_W}(\Phi^{-1}(u_1),...,\Phi^{-1}(u_n))}{\prod_{i=1}^n \phi(\Phi^{-1}(u_i))}$ where $u_i = \Phi(W_i)$ and $F^{-1}(u) = \Phi^{-1}(u)$. and $W_i = \Phi^{-1}\big(\frac{\rho_i - 1}{2}\big)$.

Hyperpriors.

$$\tau_\rho^2 \sim IG(a_1, b_1),\ \lambda_\rho \sim Uniform[0,1],\ \sigma^2 \sim IG(a_2, b_2)$$

SVAR model Full conditional distributions

Full conditional distribution of $\boldsymbol{\beta}$:

Given prior $\boldsymbol{\beta}|\tau_\beta^2 \overset{iid}{\sim} N(0, \tau_\beta^2)$, $\ [\boldsymbol{\beta}|\mathbf{Z}, \cdot] \propto [\mathbf{Z}|\mu, \boldsymbol{\beta}, \sigma^2, \boldsymbol{\rho}][\boldsymbol{\beta}|\tau_b^2]$

$\propto exp\big\{-\frac{1}{2}[\mathbf{Z} - X\boldsymbol{\beta}]'\Sigma_Z^{-1}[\mathbf{Z} - X\boldsymbol{\beta}]\big\} \times exp\big\{-\frac{1}{2\tau_\beta^2}\boldsymbol{\beta}'\boldsymbol{\beta}\big\}$

$\propto exp\big\{-\frac{1}{2}[\boldsymbol{\beta}'X'\Sigma_Z^{-1}X\boldsymbol{\beta} + \boldsymbol{\beta}'X'\Sigma_Z^{-1}\mathbf{Z} + \mathbf{Z}'\Sigma_Z^{-1}X\boldsymbol{\beta} + \frac{1}{\tau_\beta^2}\boldsymbol{\beta}'\boldsymbol{\beta}]\big\}$

by completing the square, the full conditional is $\beta|\mathbf{Z}, \cdot \sim MVN\Big(\mu_\beta, \Sigma_\beta\Big)$

$\Sigma_\beta = [X'\Sigma_Z^{-1}X + \frac{1}{\tau_\beta^2}]^{-1},\ \mu_\beta = \Sigma_\beta X'\Sigma_Z^{-1}\mathbf{Z}$

Full conditional of σ^2 :

We can write $\Sigma_Z = \sigma^2\Sigma_Z^*$

Given prior $\sigma^2|a_2, b_2 \sim IG(a_2, b_2)$, $\ [\sigma^2|\mathbf{Z}, \cdot] \propto [\mathbf{Z}|\mu, \boldsymbol{\beta}, \sigma^2, \boldsymbol{\rho}][\sigma^2|a_2, b_2]$

$\propto (\sigma^2)^{-nT/2}exp\big\{-\frac{1}{2\sigma^2}(\mathbf{Z} - \mu_Z)'{\Sigma_Z^*}^{-1}(\mathbf{Z} - \mu_Z)\big\} \times (\sigma^2)^{-a_2-1}exp\big\{\frac{-b_2}{\sigma^2}\big\}$

The full conditional distribution becomes:

$$\sigma^2|\mathbf{Z}, \boldsymbol{\beta}, \boldsymbol{\rho} \sim IG\big(\frac{nT}{2} + a_2, \frac{1}{2}(\mathbf{Z} - \mu_Z)'{\Sigma_Z'}^{-1}(\mathbf{Z} - \mu_Z) + b_2\big)$$

Full conditional of τ_ρ^2 :

We can write $\Sigma_W = \tau_\rho^2 {\Sigma_W^*}^{-1} = \tau_\rho^2[(1-\lambda_\rho)I + \lambda_\rho R]^{-1}$

Given prior $\tau_\rho^2|a_1, b_1 \sim IG(a_1, b_1)$, $\;[\tau^2|\mathbf{W}, \lambda] \propto [\mathbf{W}|\tau_\rho^2, \lambda_\rho][\tau_\rho^2|a_1, b_1]$

$\propto (\tau_\rho^2)^{-n/2} exp\big\{ -\frac{1}{2\tau_\rho^2}\mathbf{W}' {\Sigma_W^*}^{-1}\mathbf{W}\big\} \times (\tau_\rho^2)^{-a_1-1} exp\big\{\frac{-b_1}{\tau^2}\big\}$

The full conditional distribution becomes:

$$\tau_\rho^2|\mathbf{W}, \lambda_\rho \sim IG\big(\frac{n}{2} + a_1, \frac{1}{2}\mathbf{W}' {\Sigma_W^*}^{-1}\mathbf{W} + b_1\big)$$

SVAR model MH sampling algorithms

Sampling λ_ρ at iteration k :

1. Draw λ^* from proposal $q(\lambda_\rho^*|\lambda_\rho^{(k-1)}) \sim N(\lambda_\rho^{k-1}, s_\rho^2)\mathbb{1}\{0 \le \lambda_\rho^* \le 1\}$

   where $q(\lambda_\rho^*|\lambda_\rho^{(k-1)}) = \frac{\phi\big(\frac{\lambda_\rho^* - \lambda_\rho^{(k-1)}}{\tau_\lambda}\big)}{\Phi\big(\frac{1-\lambda_\rho^{(k-1)}}{s_\rho}\big) - \Phi\big(\frac{-\lambda_\rho^{(k-1)}}{s_\rho}\big)}$

2. Calculate acceptance ratio $r_\lambda = \frac{P(\lambda_\rho^*|\mathbf{Z}, \mathbf{W}^{(k-1)}, \cdot) q(\lambda_\rho^{(k-1)}|\lambda_\rho^*)}{P(\lambda_\rho^{(k-1)}|\mathbf{Z}, \mathbf{W}^{(k-1)}, \cdot) q(\lambda_\rho^*|\lambda_\rho^{(k-1)})} \propto \frac{[\mathbf{W}|\lambda_\rho^*][\lambda_\rho^*]\left[\Phi\big(\frac{1-\lambda_\rho^{(k-1)}}{s_\rho}\big) - \Phi\big(\frac{-\lambda_\rho^{(k-1)}}{s_\rho}\big)\right]}{[\mathbf{W}|\lambda_\rho^{(k-1)}][\lambda_\rho^{(k-1)}]\left[\Phi\big(\frac{1-\lambda_\rho^*}{s_\rho}\big) - \Phi\big(\frac{-\lambda_\rho^*}{s_\rho}\big)\right]}$

   $= \frac{|\Sigma_W(\lambda_\rho^*)|^{-1/2} exp\big\{-\frac{1}{2}\mathbf{W}'\Sigma_W^{-1}(\lambda_\rho^*)\mathbf{W}\big\}\left[\Phi\big(\frac{1-\lambda_\rho^{(k-1)}}{s_\rho}\big) - \Phi\big(\frac{-\lambda_\rho^{(k-1)}}{s_\rho}\big)\right]}{|\Sigma_W(\lambda_\rho^{(k-1)})|^{-1/2} exp\big\{-\frac{1}{2}\mathbf{W}'\Sigma_W^{-1}(\lambda_\rho^{(k-1)})\mathbf{W}\big\}\left[\Phi\big(\frac{1-\lambda_\rho^*}{s_\rho}\big) - \Phi\big(\frac{-\lambda_\rho^*}{s_\rho}\big)\right]}$

3. Generate $\xi \sim Uniform(0,1)$.
   $\lambda_\rho^{(k)} = \begin{cases} \lambda_\rho^* & if\ r_\lambda > \xi \text{ (with acceptance rate} = min\{1, r_\lambda\}), \\ \lambda_\rho^{(k-1)} & o.w. \end{cases}$

Sampling $\boldsymbol{\rho}$ at iteration k :

1. Draw $\boldsymbol{\rho}^*$ from proposal distribution: $q(\boldsymbol{\rho}^*|\boldsymbol{\rho}^{(k-1)}) \sim C_{\Sigma_W^{**}}(\boldsymbol{\rho}^*|\boldsymbol{\rho}^{(k-1)})$ with $\Sigma_W^{**} = {\tau_\rho^*}^2[(1-\lambda_\rho^*)I + \lambda_\rho^* R]^{-1}$ where ${\tau_\rho^*}^2$ and λ_ρ^* are proposal variance and spatial correlation parameters.

   (a) Sample $\mathbf{W}^* \sim N(\mathbf{W}^{(k-1)}, \Sigma_W^{**})$.

   (b) Transform $\boldsymbol{\rho}^* = 2\Phi(\mathbf{W}^*) - 1$.

2. Calculate acceptance ratio $r_\rho = \frac{P(\boldsymbol{\rho}^*|\mathbf{Z}, {\tau_\rho^2}^{(k-1)}, \lambda_\rho^{(k-1)}, \cdot) q(\boldsymbol{\rho}^{(k-1)}|\boldsymbol{\rho}^*)}{P(\boldsymbol{\rho}^{(k-1)}|\mathbf{Z}, {\tau_\rho^2}^{(k-1)}, \lambda_\rho^{(k-1)}, \cdot) q(\boldsymbol{\rho}^*|\boldsymbol{\rho}^{(k-1)})}$.

$$\propto \frac{[\mathbf{Z}|\boldsymbol{\rho}^*,\cdot][\boldsymbol{\rho}^*|\tau_\rho^{2(k-1)},\lambda_\rho^{(k-1)}]q(\boldsymbol{\rho}^{(k-1)}|\rho^*)}{[\mathbf{Z}|\boldsymbol{\rho}^{(k-1)}][\boldsymbol{\rho}^{(k-1)}|\tau_\rho^{2(k-1)},\lambda_\rho^{(k-1)}]q(\boldsymbol{\rho}^*|\boldsymbol{\rho}^{(k-1)})}$$

$$\propto \frac{|\Sigma_Z(\boldsymbol{\rho}^*)|^{-1/2}exp\{-\frac{1}{2}(\mathbf{Z}-\mu_Z)'\Sigma_Z^{-1}(\boldsymbol{\rho}^*)(\mathbf{Z}-\mu_Z)\}}{|\Sigma_Z(\boldsymbol{\rho}^{(k-1)})|^{-1/2}exp\{-\frac{1}{2}[\mathbf{Z}-\mu_Z]'\Sigma_Z^{-1}(\boldsymbol{\rho}^{(k-1)})[Z-\mu_Z]\}} \cdot \frac{f(\boldsymbol{\rho}^*)}{f(\boldsymbol{\rho}^{(k-1)})} \cdot \frac{q(\boldsymbol{\rho}^{(k-1)}|\boldsymbol{\rho}^*)}{q(\boldsymbol{\rho}^*|\boldsymbol{\rho}^{(k-1)})}.$$

Let $\Phi^{-1}(\frac{\boldsymbol{\rho}_i^*+1}{2}) = \mathbf{W}_i^*$ and $\Phi^{-1}(\frac{\boldsymbol{\rho}_i^{(k-1)}+1}{2}) = \mathbf{W}_i^{(k-1)}$.

Then, $f(\boldsymbol{\rho}^*) = \frac{\phi_{0,\Sigma_W}(W_1^*,\ldots,W_n^*)}{\prod_{i=1}^n \phi(W_i^*)} = \frac{exp(-\frac{1}{2}W^{*\prime}\Sigma_W^{-1}W^*)}{\prod_{i=1}^n \phi(W_i^*)}$

where $\Sigma_W = \tau_\rho^{(k-1)^2}[(1-\lambda_\rho^{(k-1)})I + \lambda_\rho^{(k-1)}R]^{-1}$.

and $q(\boldsymbol{\rho}^*|\boldsymbol{\rho}^{(k-1)}) = \frac{1}{2}g\big(\frac{\boldsymbol{\rho}^*+1}{2}\big) = \frac{\phi_{\Sigma_W}\Big[(\Phi^{-1}(\frac{\rho_1^*+1}{2}),\ldots,\Phi^{-1}(\frac{\boldsymbol{\rho}_n^*+1}{2}))-\phi_{\Sigma_W}[(\Phi^{-1}(\frac{\rho_1^{(k-1)}+1}{2}),\ldots,\Phi^{-1}(\frac{\boldsymbol{\rho}_n^{(k-1)}+1}{2}))\Big]}{2\prod_{i=1}^n \phi[\Phi^{-1}(\frac{\boldsymbol{\rho}_i+1}{2})]}$.

3. Generate $\xi \sim Uniform(0,1)$.

$$\boldsymbol{\rho}^{(k)} = \begin{cases} \boldsymbol{\rho}^* & if\ r_\rho > \xi \text{ (with acceptance rate} = min\{1, r_\rho\}), \\ \boldsymbol{\rho}^{(k-1)} & o.w. \end{cases}$$

4 Choice of hyperprior parameters

Since HIV data is often sparse, convergence issues can arise with the hyperparameters capturing variance, σ^2, τ_α^2, τ_ϕ^2, τ_δ^2 and τ_ρ^2. To address this issue we follow Bernardinelli et al. (1995a) by giving these parameters more informative Inverse Gamma hyperpriors. Specifically, we specify τ_α^2, τ_ϕ^2, τ_δ^2, $\tau_\rho^2 \sim IG(a_1, b_1)$ and $\sigma^2 \sim IG(a_2, b_2)$ with $a_1 = 5, b_1 = 2, a_2 = b_2 = 10$. Variance parameters for random effects and $\boldsymbol{\rho}$,i.e. τ_α^2, τ_ϕ^2, τ_δ^2 and τ_ρ^2, are given slightly more informative priors that σ^2 to avoid possible identifiability issues among the variance parameters.

With all three datasets we performed a small sensitivity analysis by also trying a set of less informative hyperprior parameters with $a_1 = a_2 = 1, b_1 = 0.01, b_2 = 0.1$. The more informative priors performed slightly better for Florida and California datasets while comparably with the larger New England dataset. Bernardinelli et al. (1995a) also found that a more informative prior is preferred with smaller datasets. Choice of hyperpriors did not, however, affect the comparison between models.

5 Model 3

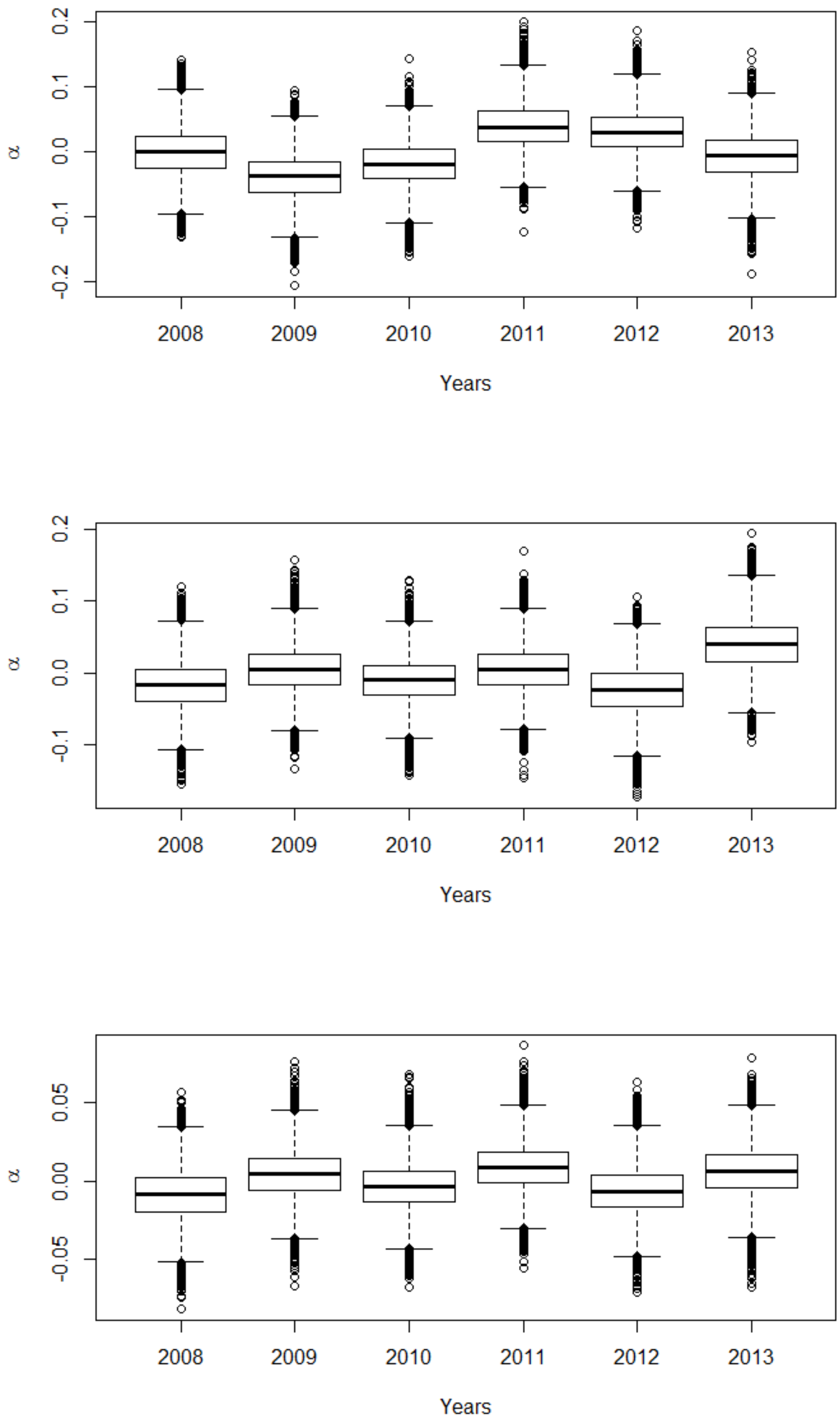

Figure 6: Boxplots of α_t for Florida, California and New England (top to bottom).